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Cognitive representation of number and understanding of place value: First graders in Croatia and the United States

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Cross-cultural comparisons in mathematics performance show differences in favor of Asian students. These results have been atributed to variations in home and school experiences. Miura examined the idea that together with educational and social influences, variability in mathematical performance may be due, in part, to differences in cognitive representation of number. Different cognitive representation of number can be affected by numerical language characteristics differentiating Asian and non-Asian language groups. Results of cross-cultural studies support that idea. This study represents an attempt to explore cognitive representation of number and place value understanding in Croatian first graders and to compare these results with data collected with U.S. first graders. The results in cognitive representations of number support Miura's idea, but our results showed no difference between groups in place value understanding.

International comparisons of mathematics achievement have consistently shown differences in favor of Asian students (Husen,1967; Lapointe, Mead & Phillips, 1989; McKnight et al., 1987). Asian children have also shown a developmental advantage in studies comparing early understanding of mathematical and pre-mathematical concepts (Mayer, Tajika & Stanley, 1991; Miura & Okamoto, 1989; Miura et al., 1993). This advantage has been seen in abstract counting ability (Miller & Stigler, 1987) and in the understanding of "Base 10" concepts (Song & Ginsburg, 1987). Song and Ginsburg found that Korean preschoolers exhibited inferior performance in informal mathematical thinking as measured by Test of Early Mathematical Ability (TEMA). This disadvantage, which they surmised might be attributable to several factors including the need to learn two counting systems, was soon overcome, and by the age of seven, Korean children in the study surpassed their U.S. counterparts in both informal and formal mathematics.

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These differences in mathematics performance and achievement have been attributed to variations in home and school experiences (Hess, Chang & McDevitt, 1987; Mordkowitz & Ginsburg, 1987; Stevenson, Lee & Stigler, 1986). Stevenson and his colleagues (1990) found differences in mathematics achievement which favored the Japanese and Chinese students in their study. They also found differences in attributions for success and failure in mathematics, in the attitudes of mothers toward schooling, and in the involvement of parents with their children in homework. Differences in the proportion of school time spent on mathematics and variations in teaching strategies were also documented (Stevenson et al., 1990; Stigler & Perry, 1988).

Miura (1987) examined the idea that, together with educational and social influences, variability in mathematics performance might also be influenced by cognitive factors. Differences in mathematical understanding and performance could be due, in part, to differences in cognitive representation of number resulting from varying numerical language characteristics that differentiate Asian and non-Asian language groups. Results of cross-national comparisons have supported that idea. For a more complete explanation, see Miura et al., (1993).

Asian languages which have their roots in ancient Chinese (Chinese, Japanese and Korean, for example) use numerical names that are congruent with "Base 10" numeration system. In this system, the value of a given digit in a multidigit numeral depends on the face value of the digit (0 through 9) and on its position in the numeral, with the value of each position increasing by powers of 10 from right to left. The spoken numerals in these Asian languages correspond exactly to their written forms. Asian children must learn names for numbers from 1 to 10. The numbers between 11 and 20 are formed as a combination of "ten" and the name for unit (e.g., 11 is spoken as "ten-one", and 12 is spoken as "ten-two"). The decad names are formed in the same way (e.g., 30 is spoken as "three-ten(s)". The generation of Asian number names is systematic and transparent (Miura & Okamoto, 1989).

Numerical names in non-Asian languages may not be congruent with the "Base 10" system. The spoken numerical names do not necessarily correspond to their written form (e.g., in English 12 is spoken as "twelve" not "ten-two"). The elements of tens and ones contained in the numeral are not apparent. English, Sweden and French children must learn number names from 1 to 20 and also the decade names. What is even more complicated in French is the change at 80 to "quatre-vingts" which means "four-twenties".

The irregularities in the English system of number words used by U.S. children promotes the use of unitary rather than multiunit conceptions for multidigit numbers, which, in turn, has implications for multidigit addition, subtraction and understanding of the place value concepts (Fuson,1990).

The Croatian language belongs to the Slavic group of languages. Slavic languages differ from each other in the construction of numerical names. For example, in Croatian, 23 is spoken as "two-tens-three" but in Slovenian, 23 is spoken as "three and two-tens". In some ways, the Croatian number language system is similar to that of Chinese, while in other ways, it is more like English (Table 1). Croatian children must learn the number names from 1 to 20; the number names from 11 to 20 are formed like teen numbers in English. Decade names, on the other hand, are formed more closely to the way they are in Asian languages. Language peculiarities, however, result in some variations in the decade names, e.g., 50 is spoken as "pedeset" which is "petdeset" (five-tens) with the "t" missing.

The purpose of the present study was to explore cognitive representation of number and place value understanding in Croatian first graders. We wished to compare these results with data collected with U.S. first graders. Miura's earlier studies (Miura, 1987; Miura et al., 1993; Miura et al., 1994) found that Asian language speakers in these studies were more likely than non-Asian language speakers to construct numbers incorporating the elements of tens and ones. Non-Asian language speakers preferred to show numbers using a collection of units. We expected that, as speakers of a language with a number counting system more similar to English than Chinese, Croatian children's cognitive representation of number and understanding of place value (as measured by our tasks) would be like that of the U.S. children. At the same time, given that the Croatian number language system has similarities to Chinese, we were curious to see if this also might have an influence on performance.

Table 1

Number Names in English, Croatian and Chinese Language

number	English	Croatian	Chinese
1	one	jedan	yi
2	two	dva	er
3	three	tri	san
4	four	etiri	si
5	five	pet	wu
6	six	šest	liu
7	seven	sedam	qi
8	eight	osam	ba
9	nine	devet	jiu
10	ten	deset	shi
11	eleven	jedanaest	shi-yi
12	twelve	dvanaest	shi-er
13	thirteen	trinaest	shi-san
14	fourteen	četrnaest	shi-si
15	fifteen	petnaest	shi-wu
16	sixteen	šesnaest	shi-liu
17	seventeen	sedamnaest	shi-qi
18	eighteen	osamnaest	shi-ba
19	nineteen	devetnaest	shi-jiu
20	twenty	dvadeset	er-shi
30	thirty	trideset	san-shi
40	forty	četrdeset	si-shi
50	fifty	pedeset	wu-shi
60	sixty	šezdeset	liu-shi
70	seventy	sedamdeset	qi-shi
80	eighty	osamdeset	ba-shi
90	ninety	devedeset	jiu-shi

METHOD

Participants

Participants in this study were 24 first graders (11 girls and 13 boys) in the United States, and 26 first graders (12 girls and 14 boys) in Croatia. The mean age of the U.S. students was 6 years and 10 months and the mean age of the Croatian children was 7 years and 3 months. The U.S. children were enrolled in a private school located near San Francisco, CA. Entrance into the school is competitive, and the curriculum is rigorous. The Croatian children were enrolled in a public school in Zagreb which serves middle-class families. In both nations, children were not selected individually for the study; entire classes participated. Cognitive representation of number was tested in the first half of the academic year in both countries. In the U.S., at the suggestion of the classroom teacher, the assessment of place-value understanding took place at the end of the school year; in Croatia, the assessment was performed two weeks after the cognitive representation of number tests. Children from both samples were monolingual in their respective languages.

Procedure

The study had two parts. In each, children were seen and tested individually in their native languages using protocols and scoring developed by Miura (Miura & Okamoto, 1989).

Part one. Cognitive representation of number was assessed using "Base 10" blocks. These blocks are used in the United States to teach place-value concepts. (However, the particular children in this study had not been introduced to the blocks prior to the testing.) These blocks are designed so that 10 unit blocks (white colored) are equivalent to 1 ten block (blue colored). The ten block is marked into 10 segments. The experimenter explained that the blocks could be used for counting and to construct numbers. The experimenter counted out 10 unit blocks one-by-one and showed the equivalence between 10 unit blocks and 1 ten block by placing them side-by-side. There were 100 unit blocks and 20 ten blocks, more than necessary for the tasks so that there were no constraints on which blocks to use. Children were asked to read a numeral written on the card and to construct the number using the blocks. There were two practice items (the numerals 2 and 7) on which coaching was permitted. Then, five items (numerals) were presented in random order: 11, 13, 28, 30, and 42. Between the two trials children were reminded of the equivalence between the two kinds of blocks. In the second trial experimenter reconstructed the child's first representation and asked if the child could construct the number using the blocks in another way.

The children's constructions were scored as correct if they summed to the whole numeral. Correct constructions were categorized separately for Trials 1 and 2 using the following categories: (a) one-to-one collection: the representation used only unit blocks (e.g., 28 unit blocks for 28); (b) canonical "Base 10" representation: the representation used the correct number of ten blocks and unit blocks (e.g., 2 ten blocks and 8 unit blocks for 28); and (c) noncanonical "Base 10" representation: the representation used some other correct number of ten blocks and unit blocks, allowing for more than 9 units (e.g., 1 ten and 18 unit blocks for 28).

Part two. In the second part of the study, children were presented with five problems designed to measure the understanding of place-value concepts. On the first item, children were shown a card with the written numeral 32. Children were asked to point to the numeral in the ones position and then to the numeral in the tens position. Each child was asked to use "Base 10" blocks to show the 3 and the 2. For the second problem, children were presented with 4 ten blocks and 4 unit blocks and asked what number the blocks made. Then, when shown the written numeral, children were asked which set of four blocks represent the 4 in the ones place and which set of four blocks represent the 4 in the tens place, and to explain how they knew that. In the third problem children were shown a non-canonical construction (3 ten blocks and 12 unit blocks) and asked to write the number the blocks made. Then, they were asked if the 4 had anything to do with how many blocks there were and if the 2 had anything to do with how many blocks there were. On the fourth item, children were given 13 unit blocks and asked to put 4 unit blocks each into three clear, plastic cups. This resulted in three cups holding four blocks each and one unit block remaining on the table. Children were asked what number the blocks made and then were shown a card with the numeral 13. They were then asked, in turn, if the 1 and the 3 had anything to do with how many blocks there were. A fifth problem was the same but used 26 blocks. These five tasks were presented in random order.

The items were scored as correct if the children's responses and justifications indicated that they understood the meaning of the individual digits in the numeral.

RESULTS

There were no gender differences on any of the variables, so students from each nation were treated as one group.

Cognitive Representation of Number

There were 130 possible constructions for the Croatian children and 120 possible constructions for the U.S. children on each trial. On Trial 1, Croatian children made 130 (100%) correct constructions; 46.2% were canonical "Base 10" constructions, 12.3% were noncanonical "Base 10" representations, and

41.5% were one-to-one collections. The U.S. children made 109 (91.0%) correct constructions; 8.3% of their correct constructions were canonical "Base 10" representations, 0.9% were noncanonical "Base 10" constructions, and 90.8% were one-to-one collections. Incorrect constructions were not categorized.

On the second trial Croatian children made 127 (97.7%) correct constructions; 33.1% of these were canonical "Base 10" representations, 28.3% were noncanonical "Base 10" representations, and 38.3% were categorized as one-to-one collections. U.S. first graders made 41 (34.2%) correct constructions; 70.7% were canonical "Base 10" representations, 19.5% were noncanonical "Base 10" constructions, and 9.8% were one-to-one collections. As for trial one, incorrect constructions were not categorized.

We used analysis of variance procedures to compare mean differences between results obtained in each of three categories. Table 2 shows means, standard deviations, and F-ratios and their probabilities for each trail separately.

Table 2

Means, standard deviations and ANOVA results for each category of cognitive representations

Category	Croatian (N=26) M	Croatian (N=26) SD	United States (N=24) M	United States (N=24) SD	F	p
Trial 1						
Canonical "Base 10"	2.31	1.74	.38	1.05	22.12	.00
Noncanonical "Base 10"	.62	.85	.04	.20	10.32	.00
One-to-one collection	2.08	1.79	4.13	1.54	18.69	.00
Trial 2						
Canonical "Base 10"	1.61	1.47	1.21	1.64	.85	.36
Noncanonical "Base 10"	1.38	1.20	.33	.76	13.37	.00
One-to-one collection	1.88	1.58	.17	.56	25.26	.00

On the first trial, U.S. first graders showed a preference for using one-to-one collection to construct numbers, and Croatian children were more likely to use either a canonical or noncanonical "Base 10" representation. On the second trial, Croatian children used more one-to-one collections and noncanonical "Base 10" representations than did U.S. children. Overall, across the two trials, there was no difference in the number of one-to-one collections used. However, Croatian students used more canonical "Base 10" representations (M=3.92) than did U.S. children (M=1.58), $F(1,48)=24.90$, $p<0.001$. The groups also differed in the overall use of noncanonical "Base 10" representations. Croatian children used more noncanonical "Base 10" representations (M=1.92) than did their U.S. counterparts (M=0.38), $F(1,48)=23.35$, $p<0.001$.

Table 3

Canonical "Base 10" representations by group across two trials

group	N	0	1	2	3	4	5
		Number of canonical "Base 10" representations					
Croatia	26	1	0	3	4	6	12
U.S.A.	24	12	2	3	0	5	2

The use of canonical "Base 10" constructions across both trials is shown in Table 3. Across the two trials, 46% of the Croatian children used a canonical "Base 10" representation to construct all five numbers, and only one child used no canonical representations at all. Across the two trials, 50% of the U.S. children did not use any canonical "Base 10" constructions, and only 8% of the U.S. students used a canonical "Base 10" construction to represent all five numbers.

Place-Value Understanding

There were five items in the place-value understanding assessment measure, and the the maximum possible score was 5. Our results showed no difference between Croatian children (M=1.12) and U.S. children (M=1.46) on these items, $F(1,48)=0.54$, $p>0.46$. The results displayed in table 4 shows that one-half of U.S children and almost one-half of Croatian children were not able to answer any of the tasks correctly. No Croatian students, and only four in the United States were able to answer all five problems correctly.

Correlational Analysis

We did not have the same measures of mathematics achievement for both samples; therefore, we could not make meaningful comparisons. We were,however, able to examine the relation between the cognitive representation of number, understanding of place value concepts, and mathematics achievement for the Croatian sample. We used student's end-of-year grades as a measure of mathematics achievement. Results in table 5 show that mathematics achievement was correlated positively with the preference for using a canonical

Table 4

Place-value understanding by group

group	N	0	1	2	3	4	5
		Number of correctly solved problems					
Croatia	26	12	5	5	2	2	0
U.S.A.	24	12	4	2	1	1	4

"Base 10" representations on Trial 1. Mathemathics achievement was also correlated negatively with the use of a one-to-one collection on the first trial.

Table 5

Correlations between cognitive representation of number, place-value understanding and mathematics achievement for the Croatian group (N=26)

representation/ place-value	Mathematics achievement
Trial 1	
Canonical "Base 10"	.52*
Noncanonical "Base 10"	.29
One-to-one collection	-.65**
Trial 2	
Canonical "Base 10"	-.14
Noncanonical "Base 10"	.24
One-to-one collection	.10
Place-value tasks	.20

*p<.01, **p<.001

DISCUSSION

The purpose of this study was to examine the cognitive representation of number (as measured by our tasks) and the understanding of place-value concepts in Croatian first graders and to compare those results with the data collected with U.S. children. Our expectation was that Croatian children's performance would differ from that of U.S. children because the number language systems of the two groups are somewhat different. Earlier studies comparing Asian and non-Asian language speakers (Miura, 1987; Miura et al., 1993) had found support for the proposition that cognitive representation of number may differ depending on the language spoken. This, in turn, might have an effect on place-value understanding and mathematics achievement. Differences between the Croatian and English number naming systems are smaller than those found between English and Asian languages, such as Chinese. However, numerical names in Croatian are clearer and more in concordance with the "Base 10" numeration system.

The results ot this study showed that cognitive representation of number appears to differ for Croatian- and English-speaking first graders. Croatian children in this study showed a preference for using canonical and noncanonical "Base 10" representations when asked to construct numbers. Assuming that these constructions are a behavioral representation of the child's mental image of number, it seems that Croatian children see numbers more like structures of tens and ones than do U.S. children. English-speaking children in this study showed an initial preference for using a collection of units to construct numbers.

Given the differences in cognitive representation of number, we also expected to find a difference in place-value understanding between the two groups. This expectation is supportable; if Croatian children view numbers as combinations of tens and units, this should strengthen their understanding of the meaning of individual digits in a multidigit numeral. However, the results showed no difference in place-value understanding between the groups. One possible explanation is that U.S. students were tested on place-value understanding at the end on the first grade when this concept might already have been introduced. At the time of testing, Croatian children had been working with addition and subtraction of numbers from 1 to 10, and the place-value concepts were unknown to them. Another explanation might be that the cognitive representation of number for the Croatian sample differs from that of the U.S. sample, but not enough to facilitate or promote earlier place-value understanding.

The correlations between cognitive representation of number and mathematics achievement were similar to those found in an earlier study (Miura, 1987). Mathemathics achievement was correlated positively with the initial preference for using a canonical "Base 10" representation to construct numbers and negatively with the use of one-to-one collections.

In addition to differences in number language characteristics, Croatian and U.S. children differ in their school experience. For example manipulative materials, such as "Base 10" blocks, and pictorial arrays for explaining place value are not used in Croatian schools. There may also be differences in parental expectations for school achievement or some other social factors to explain variability in mathematics performance (e.g., Croatian children attend kindergartens more often than U.S. children) . However, it is unlikely that these factors would affect the child's cognitive representation of number. Thus, it seems that our results support the idea that children's cognitive representation of number may be affected by the characteristics of the numerical languages they use.

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